

## THEORETICAL STUDIES OF TRANSPORT OF SOLID PARTICLES IN PULSATING FLOWS

A. I. Akhremenko, V. L. Belousov, and  
V. P. Marchenkov

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*The dynamics of the motion of solid particles in flows with a pulsation frequency of the flow rate of up to 1 Hz has been studied. Diagrams characterizing the contributions of the various components to the resultant force exerted by the fluid on the particles have been obtained.*

Pipeline hydrotransport of coal, sand, and other solid materials is constantly being improved. The use of pulsating flows of the carrier fluid, i.e., flows in which cyclic pulsations are imposed on the translational motion, is promising for increasing the efficiency of pipeline hydrotransport. Its prospects are supported by numerous studies [1-6 and others]. In particular, G. Round [3, 4] indicates that a lower mean flow velocity is required for transport of solid particles in a pulsating flow and less (by up to 50%) energy is consumed than in a steady-state flow. In Pokrovskaya's studies [6] based on experimental data, she states that in pulsating flows it is possible to reduce the mean transport rate by a factor of 1.3 to 2.0 and, consequently, to increase the distance of transport, that uses ordinary pump systems. However, lack of scientific recommendations on the choice of the flow parameters of pulsating hydrotransport, namely, amplitude and frequency, hampers wide use of it. This can be explained by the fact that the effect of these parameters on the main properties of the flow, namely, head loss, force action on the transported particles, etc., have been studied inadequately. Therefore, theoretical and experimental studies of two-phase pulsating flows are still urgent.

In the present work the effect of the pulsation frequency of a pulsating flow on the force action of the liquid on the solid fraction is studied. A low frequency range ( $f = 0-1.0$  Hz) has been chosen for the study since according to experimental observations [1-5], intensification of the work of hydrotransport is exhibited to the greatest extent in this region.

Actual curves of fluid velocities obtained in experimental studies of the process on a special laboratory stand with a diameter pipeline of 50 mm and a length of 60 m were used as initial information for mathematical models. Because of the capabilities of the stand it was possible to specify the pulsation frequency needed and the velocity of the flow in the pipeline, to charge the solid particles studied into the flow, and to carry out visual observation of the hydrotransport.

The profiles of the actual velocities of the pulsating flow obtained by measurements are *M*-shaped almost during almost the entire pulsation cycle, except for the region of developed accelerated flow  $\omega_* t = 144-216$  (Fig. 1). This indicates that the kinetic energy of the wall layers of the pulsating flow increases relative to steady-state flows, in which the velocity of the wall layers is minimum.

Features of transport of solid particles in a pulsating flow were determined using the method of test particles [7-9, 12]. The system of equations of motion of such a particle along its trajectory is written as

$$\begin{aligned} \frac{dr_p}{dt} &= v_p; \\ m_p \frac{dv_p}{dt} &= F_{lp}; \end{aligned} \quad (1)$$

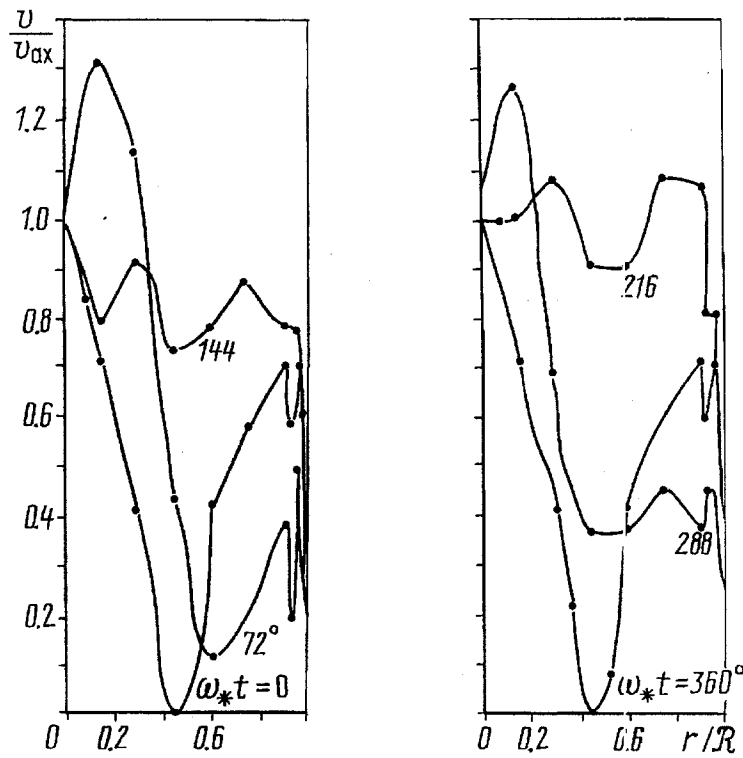


Fig. 1. Distribution of actual velocities along a pipe radius.

$$\frac{d\omega_p}{dt} = \frac{M_{lp}}{I}.$$

In the system of equations (1) the force  $F_{lp}$  can be expressed as a sum of components of various nature, including the resistance force  $F_{res}$ , the component caused by the pressure gradient in the flow  $F_p$ , the force of gravity  $F_g$ , the Archimedean buoyancy force  $F_{Ar}$ , the acceleration force of the associated mass force  $F_m$ , and the force caused by particle rotation  $F_\omega$ . Bassat force  $F_B$  and the Saffman force  $F_S$  are neglected, since the characteristic Reynolds numbers are  $Re_p \gg 1$  and  $Re_\omega \gg 1$ . Thus,

$$F_{lp} = F_{res} + F_p + F_g + F_{Ar} + F_m + F_\omega.$$

Under the assumption of a spherical shape of the particles, the components of  $F_{lp}$  are determined as follows. First,

$$F_{res} = \frac{c_D \pi d_p^2}{8} \rho |v - v_p| (v - v_p),$$

where for  $Re_p$  recorded experimentally, the particle resistance coefficient  $c_D$  is equal to [12]

$$c_D = \frac{24}{Re_p} (1 + 0.158 Re_p^{2/3}).$$

The second component is given by the relation [10]

$$F_p = \frac{\pi d_p^3}{6} \rho \frac{dv}{dt}.$$

The expression for the force of associated masses has the form [10]

$$F_m = -\frac{\pi d_p^3}{12} \rho \left( \frac{dv}{dt} - \frac{dv_p}{dt} \right).$$

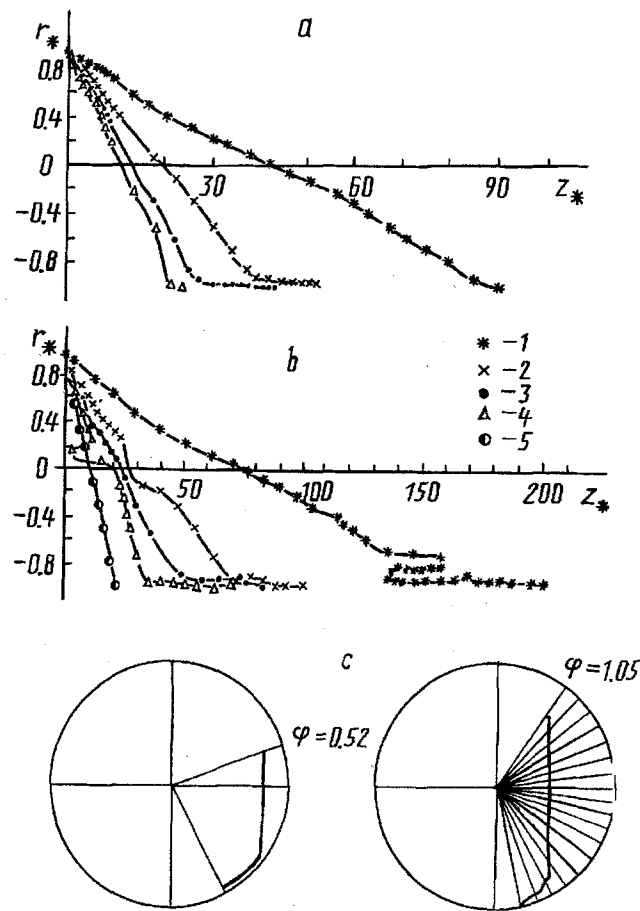


Fig. 2. Trajectories of particle motion: a) in a steady-state flow; b) in a pulsating flow ( $f = 0.45$  Hz); c) particle motion in the cross section of a pipe ( $f = 0.45$  Hz;  $d_p = 0.25$  mm); 1)  $d_p = 0.25$  mm; 2) 0.5; 3) 0.75; 4) 1.0; 5)  $d_p = 1.5$  mm.

Under the assumption of local potentiality of the flow near the particle, the force induced by rotation of the particles brought about by the velocity shear in the fluid is given by the expression [11]

$$F_\omega = \frac{4}{3} c_\omega \frac{\pi d_p^3}{8} \rho [(\mathbf{v} - \mathbf{v}_p) \times \boldsymbol{\omega}_p].$$

For the conditions assumed,  $c_\omega(\text{Re}_p, \text{Re}_\omega) = 2$  [11].

The expression for the external potential force, equal to the weight of the particle with the Archimedean force subtracted, is written as

$$F_l = F_g + F_{Ar} = \frac{\pi d_p^3}{8} (\rho - \rho_p) g.$$

In the solution of the system of equations (1) the following assumptions are made:

1. The particles do not disturb the flow field.
2. Within the region considered, external cyclic pulsations imposed on the flow are not damped.
3. The rate of propagation of pulsations in the pipeline is independent of the radial coordinate.
4. After touching the bottom, the particle loses the ability to move.

System of equations (1) is written in cylindrical coordinates. Integration of the equations of motion of test particles is carried out using the Euler method with automatic choice of the step.

In calculating the trajectories of motion the following initial conditions are used: the particles are introduced from the upper part of the cross section of the pipe; the radial and tangential components of the particle velocity

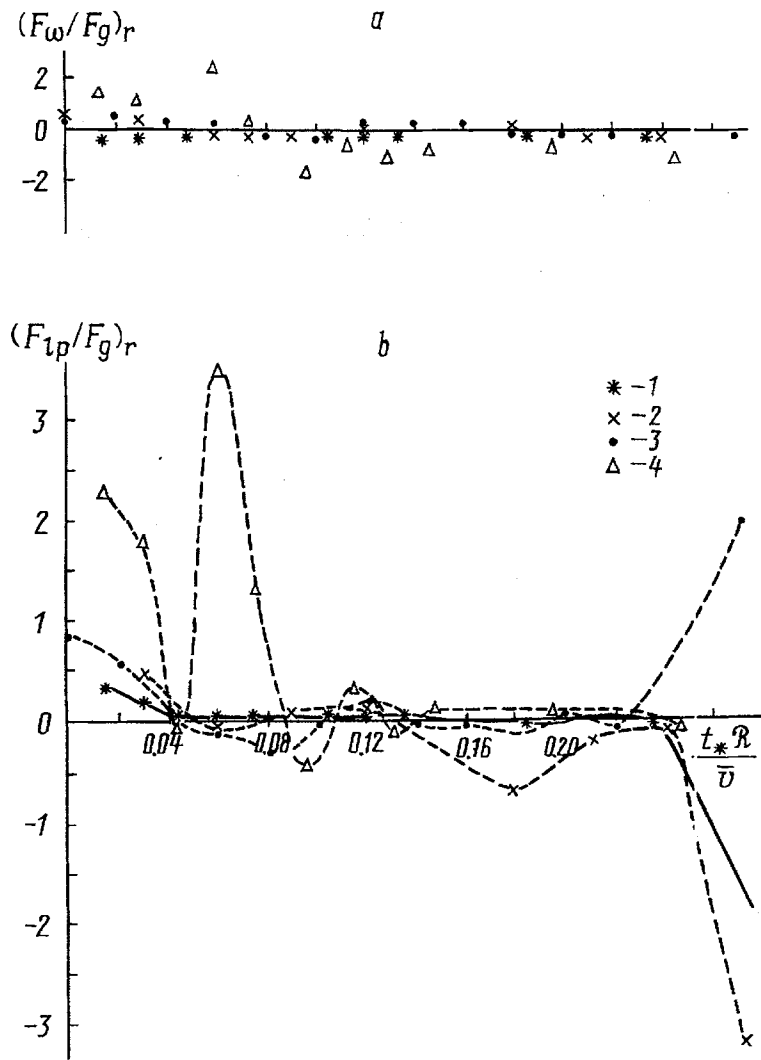


Fig. 3. Ratios of radial components of forces exerted on solid particles in a fluid: a) change induced by the pulsation frequency in the ratio of the radial components of the force caused by particle rotation and the force of gravity ( $d_p = 1$  mm); b) ratio of the radial components of the resultant force to the force of gravity ( $d_p = 1$  mm); 1)  $f = 0$  Hz; 2) 0.164; 3) 0.45; 4)  $f = 0.81$  Hz.

and the angular velocity of particle rotation are assumed to be zero; the relative particle velocity is chosen as a function of the particle diameter according to data in the technical literature for a steady-state flow [13]. The particle size is varied in the range  $d_p = 0.25-1.5$  mm, which corresponds to the actual fractional composition of sand in a hydraulic mixture. The density of the particles is  $\rho_p = 2.65$  g/cm<sup>3</sup>.

Particular trajectories of particle motion are shown in Fig. 2. Figure 2a, b characterizes motion of the particles along the pipeline for steady-state and pulsating flow of the carrier fluid, respectively, and Fig. 2c shows particle motion in the cross section of the pipeline.

The longitudinal trajectories of the particles (Fig. 2a, b) are oscillatory, which corresponds to experimental results obtained earlier by high-speed photography [14]. This is exhibited most clearly in pulsating flows (Fig. 2b). The slope of the trajectories increases with the size of the transported particles, and the length increases when pulsations are imposed on the carrier fluid, which is also consistent with experimental data [14].

It is found that the main reason for transverse migrations of particles in a pulsating flow is the action of the force of gravity and the force caused by rotation of the particles (Fig. 3). Under the action of shear in the fluid the radial component of the latter force became comparable to the force of gravity or even exceeded it in magnitude (Fig. 3a). A change in the sign of the ratio  $(F_\omega/F_g)_r$  indicates a periodic change in the direction of rotation of the

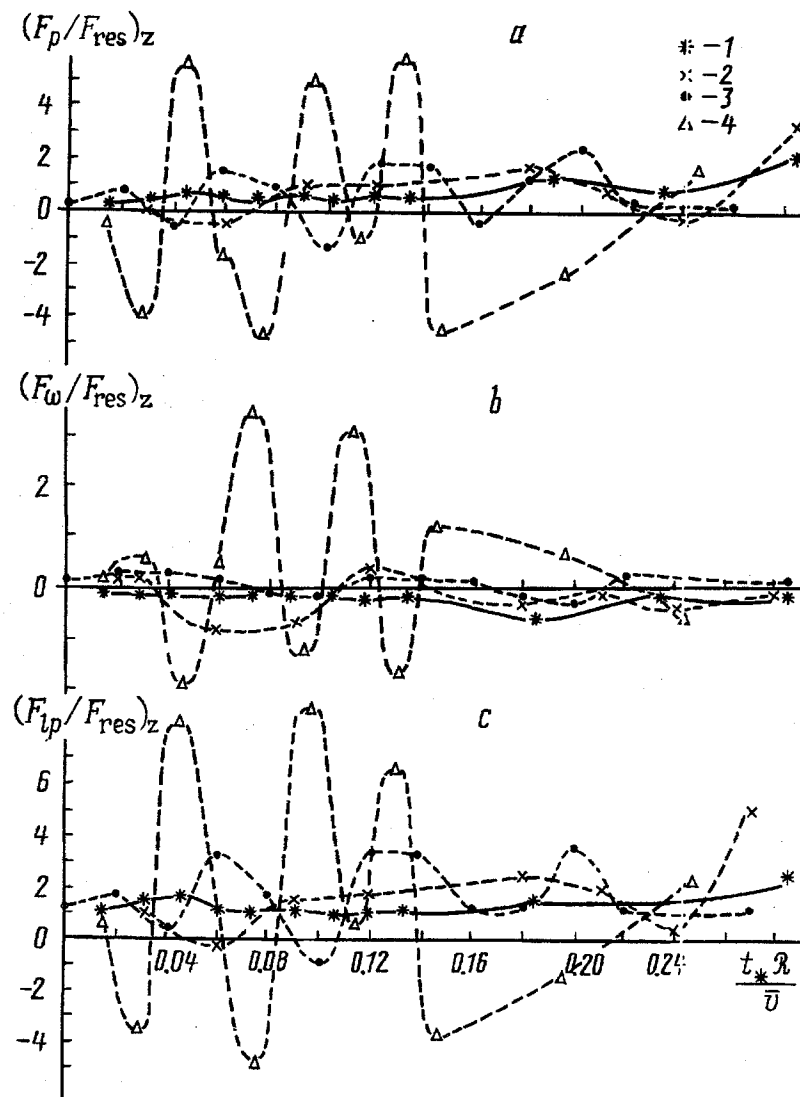


Fig. 4. Ratio of the longitudinal components to the forces exerted on solid particles in a fluid: a) ratio between the longitudinal component of the force caused by the pressure gradient and the resistance force ( $d_p = 1$  mm); b) ratio of the longitudinal component of the force caused by particle rotation to the resistance force ( $d_p = 1$  mm); c) ratio of the longitudinal component of the resultant force to the resistance force ( $d_p = 1$  mm); 1)  $f = 0$  Hz; 2) 0.164 Hz; 3) 0.45 Hz; 4) 0.81 Hz.

particles. This, in turn, determines the direction of their transverse motion, explains why the trajectories of the particles are oscillatory, and can bring about the "negative effect" [15].

The behavior of the radial component of the resultant force (Fig. 3b) indicates a decrease in the effect of the force of gravity in particle transport by a pulsating flow.

The resistance force  $F_{res}$  and the forces caused by rotation of the particles  $F_\omega$  and the pressure gradient in the carrier phase  $F_p$  have maximum components along the longitudinal coordinate (Fig. 4). The resistance force  $F_{res}$  is the main force determining the longitudinal motion of the particles in a steady-state flow. In pulsating flows the resistance force is not so important (Fig. 4c). Its contribution to the resultant force decreases to 12% in particular cases. In this case the directions of the resultant force and the resistance force can change, which gives rise to regions of reciprocating motion on the trajectories of the particles (Fig. 2b).

A decrease in the contribution of the resistance force in the pulsating flow is accompanied by an increase in the forces induced by the pressure gradient and particle rotation. This increase is more pronounced, the higher

the pulsation frequency (Fig. 4a, b). At a frequency  $f = 0.81$  Hz the forces  $F_p$  and  $F_\omega$  are several times higher than the resistance force  $F_{res}$ .

Generalization of the present results shows that in the case of transport of solid particles in pulsating flows the role of the forces caused by the pressure gradient and rotation of the particles in the shear field of velocities of the carrier fluid increases. The effect of these forces increases with the pulsation frequency in the range studied ( $f = 0-1.0$  Hz).

## NOTATION

$c_D$ , resistance coefficient of a particle;  $c_\omega$ , coefficient of transverse force;  $d_p$ , particle diameter, m;  $F_{Ar}$ , Archimedean force, N;  $F_B$ , Basset force, N;  $F_g$ , force of gravity, N;  $F_{lp}$ , resultant of all forces exerted on a particle by the fluid, N;  $F_m$ , acceleration force of the associated mass, N;  $F_p$ , force caused by the pressure gradient in the flow, N;  $F_S$ , Saffman force, N;  $F_\omega$ , force caused by particle rotation, N;  $f$ , pulsation frequency of the flow, Hz;  $g$ , gravitational acceleration,  $m/sec^2$ ;  $I$ , moment of inertia of a particle,  $kg \cdot m^2$ ;  $M_{lp}$ , moment of the resultant force relative to the rotational axis of a particle,  $N \cdot m$ ;  $m_p$ , particle mass, kg;  $R$ , inner radius of the pipe, m;  $r$ , radial coordinate, m;  $r_* = r/R$ , dimensionless radial coordinate;  $r_p$ , radius vector of particle displacement, m;  $T$ , pulsation period, sec;  $t$ , current time, sec;  $t_* = t/T$ , dimensionless time, sec;  $v$ , instantaneous linear velocity of the fluid at a certain point, m/sec;  $\bar{v}$ , average velocity of the fluid flow in a pulsation period, m/sec;  $v_{ax}$ , instantaneous axial velocity of the fluid, m/sec;  $v_p$ , linear velocity of a particle, m/sec;  $z$ , longitudinal coordinate, m;  $z_* = z/R$ , dimensionless longitudinal coordinate;  $\nu$ , kinematic viscosity,  $m^2/sec$ ;  $\rho$ , density of the fluid,  $kg/m^3$ ;  $\rho_p$ , density of a particle,  $kg/m^3$ ;  $\varphi$ , angular coordinate, rad;  $\omega_p$ , angular velocity of a particle,  $sec^{-1}$ ;  $\omega_* = 2\pi f$ , cyclic pulsation frequency,  $sec^{-1}$ . Dimensionless numbers (criteria):  $Re$ , Reynolds number ( $Re_p = 2R\bar{v}/\nu$ ,  $Re_v = v_p \cdot d_p/\nu$ ,  $Re_\omega = (0.5d_p)^2 \cdot \omega_p/\nu$ ).

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